IUHET-480
September 2004

Lorentz Violation in Supersymmetric Field Theories*

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Abstract

Broken spacetime symmetries might emerge from a fundamental physical theory. The effective low-energy theory might be expected to exhibit violations of supersymmetry and Lorentz invariance. Some illustrative models which combine supersymmetry and Lorentz violation are described, and a superspace formulation is given.

*Presented at the 3rd Meeting on CPT and Lorentz Symmetry (CPT 04), Bloomington, Indiana, 4-7 Aug 2004.

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I. INTRODUCTION

There has been an increasing realization in recent years that the Lorentz and Poincaré symmetries assumed almost universally in models of particle physics might in fact be approximate symmetries that emerge from some more fundamental theory of quantum gravity. Other potential spacetime symmetries such as supersymmetry have yet to be uncovered and would have to be broken symmetries if we are to reconcile them with experimental physics. The interesting questions then involve how is the scale of the symmetry breaking determined in each case. Viewed from the Planck scale, the scale of Standard Model symmetry breaking and electroweak-scale supersymmetry are very small. If the Lorentz symmetry is indeed broken, one of the most pressing issues would be to understand why the size of the physical effects are so incredibly tiny to have escaped all efforts to observe them experimentally. There seem to be at least some parallels in the violations of these spacetimes symmetries, it has motivated some preliminary investigations to understand any possible connection between them. Of course, since a fully solvable theory of quantum gravity is not available, the issue can not be addressed directly. Rather one must take a more phenomenological approach, allowing for all possible effects that are consistent with the remaining symmetries of the theory which might be either exact or spontaneously broken.

The approach taken in Ref. [1] is similar in spirit to the Minimal Supersymmetric Standard Model (MSSM) where supersymmetry breaking terms are added to the Standard Model where all particle fields have been expanded to include supermultiplets. Adding terms to a supersymmetric model that break the Lorentz symmetry while preserving the supersymmetry can be accomplished by modifying (deforming) the supersymmetric algebra and the supersymmetric transformation, or less generally one can leave the supersymmetric transformation unmodified. In the extensions to the Wess-Zumino model described below the supersymmetric algebra is modified, so the important issues are whether the algebra closes for the supersymmetric transformations when they are applied to the fields in the model. The approach is also in the spirit of the Standard Model Extension (SME) [2,3] where Lorentz (and CPT) violating terms are introduced into the Standard Model Lagrangian. When one adds the requirement of supersymmetry, there emerge relationships between the Lorentz-violating coefficients in a fashion similar to how masses and couplings become related in a conventional case (MSSM, for example). In Refs. [1] all possible Lorentz-violating terms were added to the Wess-Zumino model which is a theory involving only a single chiral supermultiplet. These simple models do admit a superspace formulation [4], and this motivates future systematic studies in more realistic and interesting supersymmetric models.

When supersymmetric particles are discovered at colliders, is it possible that Lorentz-violating effects could be experimentally interesting? If the effects are as suppressed as they appear to be for the observed particle content of the Standard Model, then it will be impossible to observe any new effects. In principle the Lorentz-violating effects could arise from terms in the Lagrangian involving so far unobserved superpartners to the Standard Model particles. From the point of view of phenomenology, this would mean that there are terms in the low-energy Lagrangian that violate both supersymmetry and the Lorentz symmetry [5]. These terms could be less suppressed than the analogous terms in the SME. Presumably physical effects will appear radiatively in Standard Model physics, and constraints can be derived using existing bounds.

It is well-known that the assumption of Lorentz invariance is needed in quantum field theory to avoid problems with microcausality. Field theories with Lorentz-violating terms should be regarded as effective theories and the issues involving microcausality will be addressed when the full character of the underlying fundamental theory emerges at the Planck scale [6]. While the supersymmetric theories described here should be regarded as toy models, the experimental implications of Lorentz and CPT violation parameterized in this manner have been explored extensively in recent years [7].

II. SUPERSPACE

Lorentz violation has been studied using superfields defined on superspace. Superspace is defined in terms of spacetime and superspace coordinates [8]

$$z^M = (x^\mu, \theta^\alpha, \bar{\theta}_{\dot{\alpha}}) \ , \tag{1}$$

where θ^α and $\bar{\theta}_{\dot{\alpha}}$ each form two-component anticommuting Weyl spinors. A superfield $\Phi(x,\theta,\bar{\theta})$ is then a function of the commuting spacetime coordinates x^μ and of four anticommuting coordinates θ^α and $\bar{\theta}_{\dot{\alpha}}$. A chiral superfield is a function of $y^\mu = x^\mu + i\theta\sigma^\mu\bar{\theta}$ and θ. Since the expansion in powers of θ eventually terminates this can be expanded as follows

$$\begin{aligned}
\Phi(x,\theta,\bar{\theta}) &= \phi(y) + \sqrt{2}\theta\psi(y) + (\theta\theta)\mathcal{F}(y) \ , \\
&= \phi(x) + i\theta\sigma^\mu\bar{\theta}\partial_\mu\phi(x) - \frac{1}{4}(\theta\theta)(\bar{\theta}\bar{\theta})\Box\phi(x) \\
&\quad + \sqrt{2}\theta\psi(x) + i\sqrt{2}\theta\sigma^\mu\bar{\theta}\theta\partial_\mu\psi(x) + (\theta\theta)\mathcal{F}(x) \ .
\end{aligned} \tag{2}$$

The chiral superfield can be described in terms of a differential operator U_x which is defined as

$$U_x \equiv e^{iX} \ , \tag{3}$$

where

$$X \equiv (\theta\sigma^\mu\bar{\theta})\partial_\mu \ . \tag{4}$$

Then an expansion of U_x yields

$$U_x = 1 + i(\theta\sigma^\mu\bar{\theta})\partial_\mu - \frac{1}{4}(\theta\theta)(\bar{\theta}\bar{\theta})\Box \ . \tag{5}$$

This operator effects a shift $x^\mu \to y^\mu$. Since the chiral superfield $\Phi(x,\theta,\bar{\theta})$ is a function of y^μ and θ only, the only dependence on $\bar{\theta}$ is in y^μ, so it must then be of the form $\Phi(x,\theta,\bar{\theta}) = U_x\Psi(x,\theta)$ for some function Ψ which depends only on x^μ and θ.

The first supersymmetric model with Lorentz and CPT violation involved extending the Wess-Zumino model [9]. The Wess-Zumino Lagrangian can be derived from the superspace integral

$$\int d^4\theta\Phi^*\Phi + \int d^2\theta\left[\frac{1}{2}m\Phi^2 + \frac{1}{3}g\Phi^3 + h.c.\right] \ , \tag{6}$$

where the conjugate superfield is

$$\Phi^*(x,\theta,\bar\theta) = \phi^*(z) + \sqrt{2}\bar\theta\bar\psi(z) + (\bar\theta\bar\theta)\mathcal{F}^*(z) \ , \tag{7}$$

where $z^\mu = y^{\mu *} = x^\mu - i\theta\sigma^\mu\bar\theta$. The superspace integral over $\int d^4\theta$ projects out the $(\theta\theta)(\bar\theta\bar\theta)$ component of the $\Phi^*\Phi$ superfield while the $\int d^2\theta$ projects out the $\theta\theta$ component of the superpotential. The result

$$\begin{aligned}\mathcal{L}_{WZ} = {} & \partial_\mu\phi^*\partial^\mu\phi + \frac{i}{2}[(\partial_\mu\psi)\sigma^\mu\bar\psi + (\partial_\mu\bar\psi)\bar\sigma^\mu\psi] + \mathcal{F}^*\mathcal{F} \\ & + m\left[\phi\mathcal{F} + \phi^*\mathcal{F}^* - \tfrac{1}{2}\psi\psi - \tfrac{1}{2}\bar\psi\bar\psi\right] \\ & + g\left[\phi^2\mathcal{F} + \phi^{*2}\mathcal{F}^* - \phi(\psi\psi) - \phi^*(\bar\psi\bar\psi)\right] \ ,\end{aligned} \tag{8}$$

is a Lagrangian which transforms into itself plus a total derivative under a supersymmetric transformation. The procedure just outlined is well-known and forms a basis for constructing Lorentz-violating models involving chiral superfields.

III. LORENTZ VIOLATION

Two Lorentz-violating extensions to the Wess-Zumino model were found [1], and these two models admit a superspace formulation [4]. Define new operators that can act on superfields as

$$U_y \equiv e^{iY} \ , \tag{9}$$

$$T_k \equiv e^{-K} \ . \tag{10}$$

where

$$Y \equiv k_{\mu\nu}(\theta\sigma^\mu\bar\theta)\partial^\nu \ , \tag{11}$$

$$K \equiv k_\mu(\theta\sigma^\mu\bar\theta) \ . \tag{12}$$

The expansions are

$$U_y = 1 + ik_{\mu\nu}(\theta\sigma^\mu\bar\theta)\partial^\nu - \frac{1}{4}k_{\mu\nu}k^{\mu\rho}(\theta\theta)(\bar\theta\bar\theta)\partial^\nu\partial_\rho \ , \tag{13}$$

$$T_k = 1 - k_\mu(\theta\sigma^\mu\bar\theta) + \frac{k^2}{4}(\theta\theta)(\bar\theta\bar\theta) \ . \tag{14}$$

Here $k_{\mu\nu}$ and k_μ are Lorentz-violating coefficients that transform under observer Lorentz transformations but do not transform (or transform as a scalar) under particle Lorentz transformations. They therefore represent possible descriptions of physically relevant effects. Since Y, like X, is a derivative operator, the action of U_y on a superfield $\mathcal{S}$ is a coordinate shift. The appearance of terms of order $\mathcal{O}(k^2)$ in the Lagrangians is easily understood in both cases in terms of these operators. Furthermore we have $U_y^* = U_y^{-1}$ while $T_k^* = T_k$ and not its inverse.

The supersymmetric models with Lorentz-violating terms can be expressed in terms of new superfields,

$$\Phi_y(x,\theta,\bar\theta) = U_y U_x \Psi(x,\theta) \ , \tag{15}$$
$$\Phi_y^*(x,\theta,\bar\theta) = U_y^{-1} U_x^{-1} \Psi^*(x,\bar\theta) \ . \tag{16}$$

Applying U_y to the chiral and antichiral superfields merely effects the substitution $\partial_\mu \to \partial_\mu + k_{\mu\nu}\partial^\nu$. Since U_y involves a derivative operator just as U_x, the derivation of the chiral superfield Φ_y is a function of the variables $x_+^\mu = x^\mu + i\theta\sigma^\mu\bar\theta + ik^{\mu\nu}\theta\sigma_\nu\bar\theta$ and θ analogous to how, in the conventional case, Φ is a function of the variables y^μ and θ. The Lagrangian is given by

$$\begin{aligned}
&\int d^4\theta \Phi_y^* \Phi_y + \int d^2\theta \left[\frac{1}{2} m \Phi_y^2 + \frac{1}{3} g \Phi_y^3 + h.c.\right] \\
&= \int d^4\theta \left[U_y^* \Phi^*\right] \left[U_y \Phi\right] + \int d^2\theta \left[\frac{1}{2} m \Phi^2 + \frac{1}{3} g \Phi^3 + h.c.\right] \ .
\end{aligned} \tag{17}$$

For the CPT-violating model the superfields have the form

$$\Phi_k(x,\theta,\bar\theta) = T_k U_x \Psi(x,\theta) \ , \tag{18}$$
$$\Phi_k^*(x,\theta,\bar\theta) = T_k U_x^{-1} \Psi^*(x,\bar\theta) \ . \tag{19}$$

It is helpful to note that the transformation U_x acts on Ψ and its inverse U_x^{-1} acts on Ψ^*, while the same transformation T_k acts on both Ψ and Ψ^* (since $T_k^* = T_k$). A consequence of this fact is that the supersymmetry transformation will act differently on the components of the chiral superfield and its conjugate. Specifically the chiral superfield Φ_k is the same as Φ with the substitution $\partial_\mu \to \partial_\mu + ik_\mu$ whereas the antichiral superfield Φ_k^* is the same as Φ^* with the substitution $\partial_\mu \to \partial_\mu - ik_\mu$.

The CPT-violating model can then be represented in the following way as a superspace integral:

$$\int d^4\theta \Phi_k^* \Phi_k = \int d^4\theta \Phi^* e^{-2K} \Phi \tag{20}$$

Unlike the CPT-conserving model, the $(\theta\theta)(\bar\theta\bar\theta)$ component of $\Phi^*\Phi$ no longer transforms into a total derivative. A specific combination of components of $\Phi^*\Phi$ does transform into a total derivative, and this combination is in fact the $(\theta\theta)(\bar\theta\bar\theta)$ component of $\Phi_k^*\Phi_k$.

The Lagrangians for the two models in terms of the component fields can be found in Refs. [1,4].

IV. CONCLUSIONS

Lorentz-violating extensions of supersymmetric theories model can be understood in terms of analogous transformations on modified superfields and projections arising from superspace integrals. Such superspace formulations should allow efficient investigations into possible Lorentz violation in more complicated theories.

ACKNOWLEDGMENTS

This work was supported in part by the U.S. Department of Energy under Grant No. DE-FG02-91ER40661.

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